

PARSLEY

Future Paper

Pure Mathematics - Intermediate Level

AI-generated practice paper - Limited Alpha syllabus coverage

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Time: 185 minutes Total marks: 100 Answer all questions.

This PARSLEY-generated practice paper uses only the syllabus areas currently supported by the Alpha and does not represent full syllabus coverage.

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Instructions

Show sufficient working. Each numbered question carries 10 marks. In this Alpha, labelled parts within a question are independent unless stated otherwise.

Question 1 [10 marks]

(a) The inverse of a matrix A is $A^{-1} = \begin{pmatrix} \frac{1}{2} & 0 \\ -\frac{3}{4} & -\frac{1}{2} \end{pmatrix}$. Without solving simultaneous equations, find A . [3 marks]

(b) Find $\int -4x^5 + x^4 - 5x^3 + 2 dx$. [3 marks]

(c) Given that $-\frac{6(x+2)}{(x+1)(x+3)} = -\frac{3}{x+3} - \frac{3}{x+1}$, find $\int -\frac{6(x+2)}{(x+1)(x+3)} dx$. [4 marks]

Question 2 [10 marks]

(a) The rational function $-\frac{4x-13}{(x-4)(x-3)}$ can be written as $\frac{-3}{x-(4)} + \frac{B}{x-(3)}$. Find the missing constant and hence find its indefinite integral. [5 marks]

(b) Write $-\frac{x+7}{(x-2)(x+1)}$ as $\frac{A}{x-(-1)} + \frac{B}{x-(2)}$. By substituting suitable values of x , find A and B , and hence evaluate the indefinite integral. [5 marks]

Question 3 [10 marks]

(a) Given $A = \begin{pmatrix} 2 & -1 \\ 2 & 0 \end{pmatrix}$ and $AX = \begin{pmatrix} 0 & -1 \\ 0 & 0 \end{pmatrix}$, find the matrix X by using A^{-1} . [5 marks]

(b) Find each of the following indefinite integrals: (i) $\int 4x^2 dx$; (ii) $\int -4e^x dx$; (iii) $\int \frac{3}{x} dx$, for $x > 0$. [5 marks]

Question 4 [10 marks]

(a) Given that $f'(x) = -2x^2 - 1$ and $f(-2) = \frac{28}{3}$, find $f(x)$. [4 marks]

(b) For $R(x) = \frac{kx + 3k - 3x + 9}{x^2 - 9}$, the coefficient of $\frac{1}{x - (3)}$ in its partial-fraction decomposition is -1 . Find k , then find $\int R(x) dx$. [6 marks]

Question 5 [10 marks]

(a) The matrix $A = \begin{pmatrix} k & -1 \\ -1 & -3 \end{pmatrix}$ is nonsingular. The $(2, 1)$ entry of A^{-1} is -1 . Find k and hence find A^{-1} . [4 marks]

(b) Factor the denominator of $\frac{2x + 3}{x^2 + 5x + 6}$, express the rational function in partial fractions, and hence find its indefinite integral. [6 marks]

Question 6 [10 marks]

(a) A student claims that $\int (-5x^2 - 4 \cos(x)) dx = -\frac{5x^3}{3} + 4 \sin(x) + C$. Identify the error and give the correct indefinite integral. [4 marks]

(b) Factor the denominator of $\frac{-x - 9}{x^2 - 1}$, express the rational function in partial fractions, and hence find its indefinite integral. [6 marks]

Question 7 [10 marks]

(a) Let $A = \begin{pmatrix} k & -1 \\ 2 & 3 \end{pmatrix}$. It is known that the $(2, 1)$ entry of A^{-1} is $\frac{1}{2}$. Find k , then find the $(1, 2)$ entry of A^{-1} . [4 marks]

(b) A student writes $\frac{7x+10}{(x+1)(x+2)} = \frac{3}{x+2} + \frac{4}{x+1}$ and integrates from this decomposition. Explain why the decomposition is incorrect, give the correct partial fractions, and hence find the indefinite integral. [6 marks]

Question 8 [10 marks]

(a) The function $F(x) = ke^x - \frac{2x^3}{3}$ is an antiderivative of $f(x) = -2x^2 + e^x$. Find k and hence state $\int f(x) dx$. [4 marks]

(b) A student writes $\frac{2(x+9)}{(x-3)(x+3)} = \frac{4}{x+3} - \frac{2}{x-3}$ and integrates from this decomposition. Explain why the decomposition is incorrect, give the correct partial fractions, and hence find the indefinite integral. [6 marks]

Question 9 [10 marks]

(a) Use A^{-1} , where $A = \begin{pmatrix} 3 & 1 \\ 1 & 1 \end{pmatrix}$, to solve $A \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -7 \\ -3 \end{pmatrix}$. [5 marks]

(b) Express $\frac{-4x - 7}{(x - 2)(x + 3)}$ in partial fractions and hence find its indefinite integral. [5 marks]

Question 10 [10 marks]

(a) Let $A = \begin{pmatrix} -2 & 3 \\ 4 & -4 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & x \\ 1 & \frac{1}{2} \end{pmatrix}$. Given that $AB = I$, find x and state A^{-1} . [3 marks]

(b) Exactly one of $F(x) = x^2 - e^x$ and $G(x) = x^2 + 2x - e^x$ is an antiderivative of $f(x) = 2x - e^x$. Determine which one, justify your answer, and hence state $\int f(x) dx$. [4 marks]

(c) For $A = \begin{pmatrix} -3 & 0 \\ 1 & -1 \end{pmatrix}$, exactly one of $B = \begin{pmatrix} -\frac{1}{3} & 0 \\ -\frac{1}{3} & -1 \end{pmatrix}$ and $C = \begin{pmatrix} -\frac{1}{3} & 0 \\ -\frac{1}{3} & -2 \end{pmatrix}$ is A^{-1} . Determine which matrix is the inverse, showing a suitable matrix product. [3 marks]