

# PARSLEY

## Future Paper - Solutions

### Pure Mathematics - Intermediate Level

AI-generated practice paper - Limited Alpha syllabus coverage

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Time: 185 minutes Total marks: 100 Answer all questions.

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### Question 1 [10 marks]

(a) Since  $(A^{-1})^{-1} = A$ , invert the given matrix. This gives  $A = \begin{pmatrix} 2 & 0 \\ -3 & -2 \end{pmatrix}$ . [3 marks]

Final answer:  $A = \begin{pmatrix} 2 & 0 \\ -3 & -2 \end{pmatrix}$

(b) Integrating term by term gives  $F(x) = -\frac{2x^6}{3} + \frac{x^5}{5} - \frac{5x^4}{4} + 2x + C$ . Differentiating  $-\frac{2x^6}{3} + \frac{x^5}{5} - \frac{5x^4}{4} + 2x$  gives the original integrand. [3 marks]

Final answer:  $-\frac{2x^6}{3} + \frac{x^5}{5} - \frac{5x^4}{4} + 2x + C$

(c) The rational function decomposes as  $-\frac{3}{x+3} - \frac{3}{x+1}$ . Integrating the simple fractions gives  $-3 \ln |x^2 + 4x + 3| + C$ . [4 marks]

Final answer:  $-3 \ln |x^2 + 4x + 3| + C$

### Question 2 [10 marks]

(a) The missing coefficient is  $-1$ . Thus the decomposition is  $-\frac{1}{x-3} - \frac{3}{x-4}$  and the integral is  $-3 \ln |x-4| - \ln |x-3| + C$ . [5 marks]

Final answer: missing coefficient =  $-1$ ;  $-3 \ln |x-4| - \ln |x-3| + C$

(b) The rational function decomposes as  $\frac{2}{x+1} - \frac{3}{x-2}$ . Integrating the simple fractions gives  $-3 \ln |x-2| + 2 \ln |x+1| + C$ . [5 marks]

Final answer:  $-3 \ln |x-2| + 2 \ln |x+1| + C$

**Question 3 [10 marks]**

(a) Premultiply  $AX = C$  by  $A^{-1}$ :  $X = A^{-1}C$ . Here  $A^{-1} = \begin{pmatrix} 0 & \frac{1}{2} \\ -1 & 1 \end{pmatrix}$ , hence  $X = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$ . [5

marks]

Final answer:  $X = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$

(b) (1)  $\frac{4x^3}{3} + C$ ; (2)  $-4e^x + C$ ; (3)  $3 \log(x) + C$  [5 marks]

Final answer: (1)  $\frac{4x^3}{3} + C$ ; (2)  $-4e^x + C$ ; (3)  $3 \log(x) + C$

**Question 4 [10 marks]**

(a) Integrating gives  $f(x) = -\frac{2x^3}{3} - x + C$ . Using  $f(-2) = \frac{28}{3}$  gives  $C = 2$ . Hence  $f(x) = -\frac{2x^3}{3} - x + 2$ . [4 marks]

Final answer:  $f(x) = -\frac{2x^3}{3} - x + 2$

(b) The stated coefficient gives  $k = -1$ . Then  $\frac{2(3-2x)}{x^2-9} = -\frac{3}{x+3} - \frac{1}{x-3}$ , so the integral is  $-\ln|x-3| - 3 \ln|x+3| + C$ . [6 marks]

Final answer:  $k = -1$ ;  $-\ln|x-3| - 3 \ln|x+3| + C$

**Question 5 [10 marks]**

(a) Using the  $2 \times 2$  inverse formula,  $(A^{-1})_{21} = -\frac{1}{3k+1}$ . Equating this to  $-1$  gives  $k = 0$ . Thus  $A^{-1} = \begin{pmatrix} 3 & -1 \\ -1 & 0 \end{pmatrix}$ . [4 marks]

Final answer:  $k = 0$ ,  $A^{-1} = \begin{pmatrix} 3 & -1 \\ -1 & 0 \end{pmatrix}$

(b) The rational function decomposes as  $\frac{3}{x+3} - \frac{1}{x+2}$ . Integrating the simple fractions gives  $-\ln|x+2| + 3 \ln|x+3| + C$ . [6 marks]

Final answer:  $-\ln|x+2| + 3 \ln|x+3| + C$

**Question 6 [10 marks]**

(a) The proposal differentiates to  $-5x^2 + 4 \cos(x)$ , not the integrand. The correct result is  $-\frac{5x^3}{3} - 4 \sin(x) + C$ . [4 marks]

Final answer:  $-\frac{5x^3}{3} - 4 \sin(x) + C$

(b) The rational function decomposes as  $\frac{4}{x+1} - \frac{5}{x-1}$ . Integrating the simple fractions gives  $-5 \ln|x-1| + 4 \ln|x+1| + C$ . [6 marks]

Final answer:  $-5 \ln|x-1| + 4 \ln|x+1| + C$

**Question 7 [10 marks]**

(a) From the inverse formula, the given entry equals  $-\frac{2}{3k+2}$ . Setting this to  $\frac{1}{2}$  gives  $k = -2$ . The required entry is then  $-\frac{1}{4}$ . [4 marks]

Final answer:  $k = -2$ ; required entry =  $-\frac{1}{4}$

(b) The proposed decomposition recombines incorrectly. The correct decomposition is  $\frac{4}{x+2} + \frac{3}{x+1}$ , giving  $3 \ln |x+1| + 4 \ln |x+2| + C$ . [6 marks]

Final answer:  $3 \ln |x+1| + 4 \ln |x+2| + C$

**Question 8 [10 marks]**

(a) Differentiating the proposed antiderivative and matching  $f(x)$  gives  $k = 1$ . Hence

$$\int f(x) dx = -\frac{2x^3}{3} + e^x + C. \quad [4 \text{ marks}]$$

Final answer:  $k = 1$ ;  $-\frac{2x^3}{3} + e^x + C$

(b) The proposed decomposition recombines incorrectly. The correct decomposition is  $-\frac{2}{x+3} + \frac{4}{x-3}$ , giving  $4 \ln |x-3| - 2 \ln |x+3| + C$ . [6 marks]

Final answer:  $4 \ln |x-3| - 2 \ln |x+3| + C$

**Question 9 [10 marks]**

(a)  $A^{-1} = \begin{pmatrix} \frac{1}{2} & -\frac{1}{2} \\ -\frac{1}{2} & \frac{3}{2} \end{pmatrix}$ . Therefore  $\begin{pmatrix} x \\ y \end{pmatrix} = A^{-1} \begin{pmatrix} -7 \\ -3 \end{pmatrix} = \begin{pmatrix} -2 \\ -1 \end{pmatrix}$ . [5 marks]

Final answer:  $x = -2, y = -1$

(b) The rational function decomposes as  $-\frac{1}{x+3} - \frac{3}{x-2}$ . Integrating the simple fractions gives  $-3 \ln |x-2| - \ln |x+3| + C$ . [5 marks]

Final answer:  $-3 \ln |x-2| - \ln |x+3| + C$

**Question 10 [10 marks]**

(a) Since  $AB = I$  and  $A$  is nonsingular,  $B = A^{-1}$ . Computing the inverse gives  $A^{-1} = \begin{pmatrix} 1 & \frac{3}{4} \\ 1 & \frac{1}{2} \end{pmatrix}$ , so

$x = \frac{3}{4}$ . [3 marks]

Final answer:  $x = \frac{3}{4}, A^{-1} = \begin{pmatrix} 1 & \frac{3}{4} \\ 1 & \frac{1}{2} \end{pmatrix}$

(b) Differentiate both candidates. Only  $A$  differentiates to  $f(x)$ , so  $\int f(x) dx = x^2 - e^x + C$ . [4 marks]

Final answer:  $A; \int f(x) dx = x^2 - e^x + C$

(c) Multiply by  $A$ . We obtain  $AB = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$  and  $AC = \begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix}$ . Therefore  $B$  is  $A^{-1}$ . [3 marks]

Final answer:  $B = A^{-1} = \begin{pmatrix} -\frac{1}{3} & 0 \\ -\frac{1}{3} & -1 \end{pmatrix}$